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BRIEF COMMUNICATION

ON SOO'S EQUATIONS IN MULTIDOMAIN MULTIPHASE FLUID MECHANICS

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In a recent article, Sha & Soo (1978) present a set of equations for the general mixture conservation equations. The purpose of this note is to show that their model is not even valid in one-dimensional situations.

The general mixture conservation equation is presented by Sha & Soo as

$$\frac{\partial}{\partial t}\rho_m\psi_m + \nabla\cdot(\mathbf{U}_m\rho_m\psi_m) = -\nabla\cdot\mathbf{J}_m + \rho_m\phi_m. \qquad [1]$$

Components of phases k relate to the mixture according to

$$\rho_m = \sum_k \rho_k$$
$$\rho_m\mathbf{U}_m = \sum_k \rho_k\mathbf{U}_k \qquad [2]$$
$$\rho_m\psi_m = \sum_k \rho_k\psi_k$$
$$\mathbf{J}_m = \sum_k \mathbf{J}_{mk}.$$

They thought that the basic conserved quantities, $\rho_m\psi_m$ are transferred with the velocity of the center-of-mass, $\mathbf{U}_m$ across the surface of a control volume. Of course the mixture mass flux is transferred with $\mathbf{U}_m$ across the surface of a control volume. But the mixture momentum and energy flux should be transferred with the velocity of the center-of-momentum, $\mathbf{U}_p$ and of the center-of-energy, $\mathbf{U}_e$ across the surface of a control volume, respectively. This concept is well illustrated by Lahey & Moody (1977). Here the velocity of the center-of-mass, $\mathbf{U}_m$ in the two phase system is equivalent to the velocity of propagation of the plane through which no net mass flux passes. By equating the mass flux terms

$$\rho_G\alpha(U_G - U_m) = \rho_L(1-\alpha)(U_m - U_L)$$

which produces

$$U_m = \frac{\rho_L U_L(1-\alpha) + \rho_G U_G\alpha}{\rho_L(1-\alpha)+\rho_G\alpha} = \frac{G}{\bar{\rho}} \qquad [3]$$

where

$$G = \rho_L U_L(1-\alpha) + \rho_G U_G\alpha$$
$$\bar{\rho} = \rho_L(1-\alpha) + \rho_G\alpha.$$

Similarly, by equating the momentum flux terms to define the plane through which zero net momentum flux passes,

$$\rho_G \alpha U_G(U_G - U_p) = \rho_L(1-\alpha)U_L(U_p - U_L)$$

which produces

$$U_p = \frac{\rho_G \alpha U_G^2 + \rho_L(1-\alpha)U_L^2}{\rho_G \alpha U_G + \rho_L(1-\alpha)U_L} = \frac{G}{\rho'}, \qquad [4]$$

where

$$\frac{1}{\rho'} = \frac{(1-x)^2}{\rho_L(1-\alpha)} + \frac{x^2}{\rho_G \alpha}.$$

Finally, by defining the plane through which no net energy flux passes

$$\rho_G \alpha e_G(U_G - U_e) = \rho_L(1-\alpha)e_L(U_e - U_L)$$

which yields

$$U_e = \frac{\rho_G \alpha e_G U_G + \rho_L(1-\alpha)e_L U_L}{\rho_G \alpha e_G + \rho_L(1-\alpha)e_L} = \frac{Ge}{\bar{e}\bar{\rho}}, \qquad [5]$$

where

$$e = e_L + x e_{LG}$$

and

$$\bar{e} = [\rho_G \alpha e_G + \rho_L(1-\alpha)e_L]/\bar{\rho}.$$

Therefore $\mathbf{U}_m$ should be replaced by $\mathbf{U}_p$ for the momentum equations and by $\mathbf{U}_e$ for the energy equations. It is easy to prove this relationship

$$\frac{\partial}{\partial t}\rho_m \psi_m + \nabla \cdot (\mathbf{U}_i \rho_m \psi_m) = \frac{\partial}{\partial t}[\rho_L(1-d)\psi_L + \rho_G \alpha \psi_G] + \nabla \cdot [\rho_L(1-\alpha)U_L \psi_L + \rho_G \alpha U_G \psi_G], \qquad [6]$$

where $U_i = U_m$ for the continuity equations; $U_i = U_p$ for the momentum equations; $U_i = U_e$ for the energy equations.

Now let us consider the conditions in which Sha & Soo's model is valid. Sha & Soo's model considers that $\mathbf{U}_m = \mathbf{U}_p = \mathbf{U}_e$. In order to satisfy the above conditions $\bar{\rho}$ is equal to ρ' from [3] and [4].

$$\frac{\bar{\rho}}{\rho'} = 1 = [(1-\alpha)\rho_L + \alpha\rho_G]\left[\frac{(1-x)^2}{\rho_L(1-\alpha)} + \frac{x^2}{\rho_G \alpha}\right] = \left[\frac{(1-x)S + x}{U_L S(1-x) + U_G x}\right]\left[\frac{U_L(1-x)^2}{(1-\alpha)} + \frac{U_G x^2}{\alpha}\right]. \qquad [7]$$

If $\alpha = x$ and slip ratio, $S = 1$, [7] is satisfied. By $\alpha - x$ relation,

$$\alpha = \frac{x}{x + S\left(\frac{\rho_G}{\rho_L}\right)(1-x)} \qquad [8]$$

ρ_G should be equal to ρ_L in order to satisfy the above conditions, $\alpha = x$ and $S = 1$. The above result comes out of their postulate that the conservation equations for the mixture of a multiphase system assume the same form as those for a homogeneous medium. It is clear that a multiphase system is different from a homogeneous medium such as an air system.

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